Written Exam for the M.Sc. in Economics autumn 2012-2013

Contract Theory

Final Exam / Master's Course

January 7, 2013

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

Attempt both questions

Question 1 (adverse selection)

The following is a model of a monopoly insurance market with adverse selection and with is a tax on the insurance premium. It builds on the standard adverse selection model that we studied in the course.

The principal (P) is a monopoly insurance company and the agent (A) is a car owner who may want to take a car insurance. Depending on how skillful A is as a driver, she may or may not have an accident. The probability of having an accident depends on A's type. A skillful (and therefore a *low*-demand) driver has an accident with probability $\underline{\theta}$, and a less skillful (and therefore a *high*-demand) driver has an accident with probability $\overline{\theta}$. Assume that $0 < \underline{\theta} < \overline{\theta} < 1$.

A's disutility of having an accident, measured in monetary terms as a deduction from her income, is denoted by $d > 0$, and A's monetary income is denoted by $w > d$. Moreover, A's payment *to* P, the premium, is denoted by p; and the compensation A receives from P in case there is an accident is denoted by a. A is risk averse and her utility function is denoted by u (where $u' > 0$ and $u'' < 0$). The government imposes a tax τ on the insurance. This means that for any given premium p, A must actually pay $(1+\tau)\,p$. The tax revenues enter the government's budget (which is not modeled here); therefore, out of the amount paid by A, only p adds to the insurance company's profit. We assume that the tax rate is positive but not too large:

$$0 < \tau < \frac{1-\overline{\theta}}{\overline{\theta}}.$$

A's utility if purchasing the insurance is

$$\begin{cases} u\left[w-(1+\tau)\,p-d+a\right] & \text{if having an accident} \\ u\left[w-(1+\tau)\,p\right] & \text{if not having an accident.} \end{cases}$$

P does not know the type of A, but assigns the probability $\upsilon \in (0,1)$ to the event that $\theta = \underline{\theta}$.

P offers a menu of two distinct contracts to A. As in the course, the contract variables are indicated either with "upper-bars" or "lower-bars", depending on which type the contract is aimed at. The contract variables are p and a. However, to solve the problem it is more convenient to think of P as choosing the utility levels directly, instead of the contract variables. Thus introduce the following notation:

$$\overline{u}_N \equiv u\left[w-(1+\tau)\,\overline{p}\right], \quad \overline{u}_A \equiv u\left[w-(1+\tau)\,\overline{p}-d+\overline{a}\right],$$

$$\underline{u}_N \equiv u\left[w-(1+\tau)\,\underline{p}\right], \quad \underline{u}_A \equiv u\left[w-(1+\tau)\,\underline{p}-d+\underline{a}\right].$$

Also let h be the inverse of u (hence $h' > 0$ and $h'' > 0$). Given that P is risk neutral and wants to maximize its expected profit, P's objective function can be written as

$$\begin{aligned} V &= \upsilon\left[\underline{p}-\underline{\theta a}\right]+(1-\upsilon)\left[\overline{p}-\overline{\theta a}\right] \\ &= \frac{\upsilon}{1+\tau}\left[w-\underline{\theta}(1+\tau)d-\left[1-\underline{\theta}(1+\tau)\right]h\left(\underline{u}_N\right)-\underline{\theta}(1+\tau)h\left(\underline{u}_A\right)\right] \\ &\quad +\frac{1-\upsilon}{1+\tau}\left[w-\overline{\theta}(1+\tau)d-\left[1-\overline{\theta}(1+\tau)\right]h\left(\overline{u}_N\right)-\overline{\theta}(1+\tau)h\left(\overline{u}_A\right)\right]. \end{aligned}$$

P wants to maximize V w.r.t. $(\underline{u}_N, \underline{u}_A, \overline{u}_N, \overline{u}_A)$, subject to the following four constraints:

$$\left(1-\overline{\theta}\right)\overline{u}_N+\overline{\theta}\overline{u}_A \geq \overline{U}^*, \qquad \text{(IR-high)}$$

$$\left(1-\underline{\theta}\right)\underline{u}_N+\underline{\theta}\underline{u}_A \geq \underline{U}^*, \qquad \text{(IR-low)}$$

$$\left(1-\overline{\theta}\right)\overline{u}_N+\overline{\theta}\overline{u}_A \geq \left(1-\overline{\theta}\right)\underline{u}_N+\overline{\theta}\underline{u}_A, \qquad \text{(IC-high)}$$

$$\left(1-\underline{\theta}\right)\underline{u}_N+\underline{\theta}\underline{u}_A \geq \left(1-\underline{\theta}\right)\overline{u}_N+\underline{\theta}\overline{u}_A, \qquad \text{(IC-low)}$$

where

$$\overline{U}^* \equiv \left(1-\overline{\theta}\right)u(w)+\overline{\theta}u(w-d), \quad \underline{U}^* \equiv \left(1-\underline{\theta}\right)u(w)+\underline{\theta}u(w-d)$$

are the two types' outside options.

a) Suppose $\tau = 0$ and that P can observe A's type. Then both types will be offered a contract with full insurance (so that $\overline{u}_N = \overline{u}_A$ and $\underline{u}_N = \underline{u}_A$). Explain, in words, the economic logic behind this result.

b) Prove that IC-low and IC-high jointly imply the following:

- If the high-demand type is underinsured ($\overline{u}_N > \overline{u}_A$), so is the low-demand type ($\underline{u}_N > \underline{u}_A$).

c) Argue, in words, that the second-order condition to P's problem is satisfied.

d) Assume that the constraints (IR-high) and (IC-low) are lax at the second-best optimum (so that they can be disregarded). Solve P's problem and characterize the optimal second-best utility levels, $\overline{u}_N^{SB}, \overline{u}_A^{SB}, \underline{u}_N^{SB}, \underline{u}_A^{SB}$. Will the low- and high-demand type, respectively, be underinsured, fully insured or overinsured at the second-best optimum?

Question 2 (moral hazard)

Consider the following moral hazard model with mean-variance preferences that we studied in the course. There is one (single) agent, A, and one principal, P. A chooses an effort level $e \in \Re_+$, thereby incurring the cost $c(e) = \frac{1}{2}e^2$. Given a choice of e, the output (i.e., A's performance) equals $q = e + z$, where z is an exogenous random term drawn from a normal distribution with mean zero and variance ν. It is assumed that P can observe q but not e. Moreover, neither P nor A can observe z. A's wage (i.e., the transfer from P to A) can only be contingent on the output q. It is restricted to be linear in q:

$$t = \alpha + \beta q = \alpha + \beta (e + z) .$$

A is risk averse and has a CARA utility function: $U = -\exp[-r(t - c(e))]$, where r (> 0) is the coefficient of absolute risk aversion. Therefore A's expected utility is

$$EU = -\int_{-\infty}^{\infty} \exp[-r(t - c(e))] f(z)\, dz,$$

where $f(z)$ is the density of the normal distribution. P's objective function is

$$V = q - t = q - \alpha - \beta q = (1 - \beta)(e + z) - \alpha,$$

which in expected terms becomes $EV = (1 - \beta) e - \alpha$. It is also assumed that A's outside option utility is $\widehat{U} = -\exp\left[-r\widehat{t}\right]$, where $\widehat{t} > 0$. The timing of events is as follows.

1. P chooses the contract parameters, α and β.
2. A accepts or rejects the contract and, if accepting, chooses an effort level.
3. The noise term z is realized and A and P get their payoffs.

Answer the following questions:

a) Solve for the β-parameter in the second-best optimal contract, denoted β^{SB} (you do not need to solve for α^{SB}, and you will not get any credit if you nevertheless do that). You should make use of the following (well-known) result:

$$EU = -\exp\left[-r\left(\alpha + \beta e - \frac{1}{2}e^2 - \frac{1}{2}\nu r \beta^2\right)\right].$$

[You are encouraged to attempt parts b), c) and d) even if you have not been able to answer parts a).]

b) Does the agent get any rents at the second-best optimum? Do not only answer yes or no, but also explain how you can tell.

c) The first-best values of the effort level and the β-parameter equal $e^{FB} = 1$ and $\beta^{FB} = 0$, respectively. How do these values relate to the corresponding second-best values? In particular, is there under- or overprovision of effort at the second-best optimum?

d) Consider the limit case where $r \to 0$. Explain what happens to the relationship between the second-best and the first-best effort levels. Also explain the intuition for this result.

END OF EXAM